SOLUTION OF ONE PROBLEM OF CONTROL CONNECTED WITH THE REDUCTION OF IRON-ORE PELLETS

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UDC 621.78.012.5:536.244

The article examines the mathematical model of the process of heating iron-ore pellets, and a gas blown through them, by high-frequency currents. On the basis of the solution of the problem of control it becomes possible to evaluate the efficiency of the heater and to choose the optimum operating conditions for it.

1. The technology of reducing iron-ore pellets to iron has been widely investigated in recent years; this technology is based on high-frequency currents for heating to very high temperatures a medium of two components, the pellets and the reducing gas blown through them [1].

On the basis of a mathematical model the present article solves the problem of determining the current intensity in the inductor ensuring the specified heating regime. The solution of this problem of control, obtained by methods analogous to those of [2, 3], makes it possible to automate the procedure of the mathematical modeling of the process on a computer. We present the results of the calculation of the heating in dependence on the geometric and dynamic parameters, on the basis of which an evaluation of the optimum regime can be obtained.

2. The technological process can be carried out in an installation whose geometric configuration is shown in Fig. 1. The installation consists of a central "active" zone of cylindrical shape in which hydrogen and the charge move in opposite directions, the charge consisting of iron-ore pellets. We denote the speed of the charge v_c, the speed of the gas v_g. This zone is encircled by an inductor through which the high-frequency current $Ie^{i\omega t}$ flows. The entire structure is enclosed in a jacket whose purpose is shielding.

The electromagnetic field induced by the high-frequency currents penetrates into the "active" zone and gives rise to Foucauld currents. These currents cause the pellets to become hot, and they in turn transmit their heat to the gas that is blown through. We point out that the installation in question may be used both for heating gas (here it is expedient to put $v_c = 0$) and for reducing pellets to pure iron.

In the formulation of the mathematical model of heating we use spatially averaged formulas for the effective electromagnetic and thermophysical parameters of the medium [1], and also experimental data, taking into account the dependence of all the parameters on the temperature. Thus we will regard both phases of the medium as "interpenetrating."

Here c_c, ρ_c, c_g, and ρ_g are the heat capacities and densities of the solid porous phase and the gas, respectively; u and w are their respective temperatures; k is the effective thermal conductivity of the examined medium; ε is a coefficient of the porous solid phase. We neglect the thermal conduction of the gas compared with the heat transfer, and then the steady thermophysical process is described by the following system of nonlinear equations in cylindrical coordinates:

$$v_c(1-\varepsilon(r))\,c_c(u)\,\rho_c(u)\frac{\partial u}{\partial z}-\alpha_V(v_g(\varepsilon))(u-w)+\frac{1}{r}\frac{\partial}{\partial r}\left(rk(u)\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial z}\left(k(u)\frac{\partial u}{\partial z}\right)+q=0,$$

$$-v_g(\varepsilon)\,\varepsilon(r)\,c_g(w)\,\rho_g(w)\frac{\partial w}{\partial z}+\alpha_V(v_g(\varepsilon))(u-w)=0,$$

$$w|_{z=0}=w_g^0;\quad \lim_{r\to 0} rk(u)\frac{\partial u}{\partial r}=0, \tag{1}$$

M. V. Lomonosov Moscow State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 6, pp. 971-977, December, 1984. Original article submitted August 3, 1983.

0022-0841/84/4706-1449$08.50

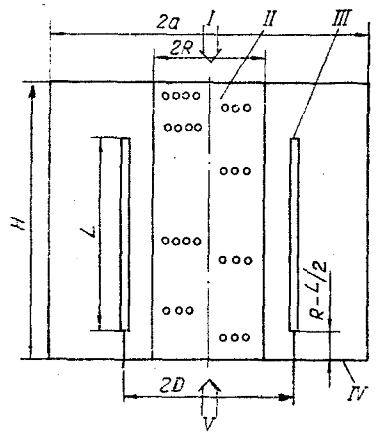


Fig. 1. Geometric configuration of the installation: I) charging; II) the charge and the gas blown through it; III) inductor; IV) jacket; V) gas being blown through.

$$-k(u)\frac{\partial u}{\partial r}\bigg|_{r=R} = \varkappa(u-u_0) + \delta\sigma(u^4 - u_0^4),$$

$$u|_{z=H} = u_c^0 \text{ for } v_c \neq 0, \quad \frac{\partial u}{\partial z}\bigg|_{z=H} = 0 \text{ for } v_c = 0.$$

Here, α_V is the coefficient of mutual heat transfer between the phases, depending on the speed of the gas, $W/m^3 \cdot {}^\circ C$, $\varkappa$ is the coefficient of convective heat transfer with the environment, $W/m^2 \cdot {}^\circ C$; σ is the Stefan–Boltzmann constant, the coefficient of ideal radiative heat exchange, $W/m^2 \cdot {}^\circ C^4$; δ is the degree of blackness of the body.

The density of the heat sources is expressed by the formula $q = 0.5\lambda(u)|E|^2$, where $\lambda(u)$ is the effective conductivity of the medium, and E, the sole nonzero angular component of the amplitude of the electric field $\mathbf{E} = \{0, E, 0\}$, is determined by the associated system of equations:

$$\begin{gathered}
\frac{\partial}{\partial r}\left(\frac{1}{r\mu(u)}\frac{\partial}{\partial r}(rE)\right) + \frac{\partial}{\partial z}\left(\frac{1}{\mu(u)}\frac{\partial E}{\partial z}\right) - i\omega\mu_0\lambda(u)E = 0, \quad 0 < r < R, \\
E|_{r=R-0} = E|_{r=R+0} + E^0(R, z), \\
\frac{1}{\mu(u)}\frac{\partial}{\partial r}(rE)|_{r=R-0} = \frac{\partial}{\partial r}(rE)|_{r=R+0} - i\omega\mu_0 R H_z^0(R, z), \\
\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rE)\right) + \frac{\partial^2 E}{\partial z^2} = 0, \quad R < r < a, \\
E|_{r=a} = -E^0(a, z), \\
E|_{z=0} = -E^0(r, 0), \; E|_{z=H} = -E^0(r, H), \; R < r < a, \\
E|_{z=0,H} = 0, \quad 0 < r < R, \\
E|_{r=0} = 0.
\end{gathered} \tag{2}$$

Here, $\mu = \mu(u)$ is the effective relative magnetic permeability of the active medium; E^0 and H_z^0 are the angular and the axial components, respectively, of the electromagnetic field created by the inductor in the free space with the specified current intensity I. We point out that the boundary conditions on the surface of the jacket correspond to ideal shielding of the working region.

The field of the inductor — a terminal solenoid — permits the explicit expression of [4] which we used in the calculations. (The inductor was approximated by coaxially arranged turns with current I.)

3. The problem (1)-(2) determines algorithmically the temperature and electrical fields for any specified current in the inductor. The corresponding algorithms are based on the theory of difference schemata [5]. The differential operators with the associated boundary conditions contained in (1), (2) are approximated on appropriately chosen grids by the difference operators M and N with second order of accuracy.

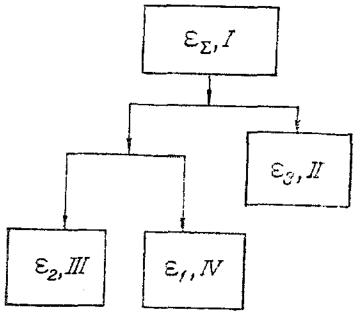


Fig. 2. Diagram of the energy balance in steady-state regime: I) introduced energy; II) losses in the inductor; III) heat losses via the wall; IV) useful energy of the heated gas.

The solution of the nonlinear system (1)-(2) was found in an iteration cycle analogous to [6, 7]: $E^{(s+1)}$: $N[E, u^{(s)}, w^{(s)}] = 0$, $u^{(s+1)}, w^{(s+1)}$: $M[u, w, E^{(s+1)}] = 0$, $u^{(0)} \equiv 0$, $w^{(0)} \equiv 0$, s = 0, 1, ... The system of difference equations $M[u, w, E^{(s+1)}] = 0$ was solved at each step of the iteration process by Siedel's method; the system $N[E, u^{(s)}, w^{(s)}] = 0$ was solved by the method of simple iteration [8]. The method of simple iterations was used in view of the complex nature of the matrix of the difference system of equations approximating (2).

4. As object of the control of the process we introduce the requirement that the temperature of the heated gas at the outlet from the system be close to the specified temperature $u_H = 950°C$. The thermal state of the gas at the outlet will be characterized by the mean temperature:

$$\bar{w} = 2\pi \int_0^R w|_{z=H} r\,dr \quad (\pi R^2)^{-1}.$$

The value $\bar{w} = \bar{w}(I)$ is algorithmically determined by the difference operator (1)-(2) and the quadrature formula of rectangles for each current intensity I and set of other parameters of the system. Then the inverse problem of control reduces, analogously to [2, 3], to the selection of values of I from the set of practical equivalence:

$$|\bar{w}(I) - u_H| \leqslant \delta, \tag{3}$$

where δ is the specified allowance. Since $\bar{w}(I)$ is continuous and monotonic, this problem is well posed in the extended sense [3].

I is sought by the method of chords for the equation $\bar{w}(I) = u_H$ with the iteration discontinued when the specified accuracy δ is attained [7]. The initial approximation is chosen in analogy to [2, 3] with the aid of the formula

$$I^{(1)} = 0.5 I^{(0)} [1 + (u_H - \bar{w}(0))/(\bar{w}(I^{(0)}) - \bar{w}(0))],$$

where $I^{(0)}$ may be specified arbitrarily in accordance with the order of the sought magnitude. We point out that for solving the problem (3) with $\delta = 40°C$, 2-4 iterations suffice, as a rule.

5. Let us now evaluate the effectiveness of the investigated system when it is a gas heater ($v_c = 0$).

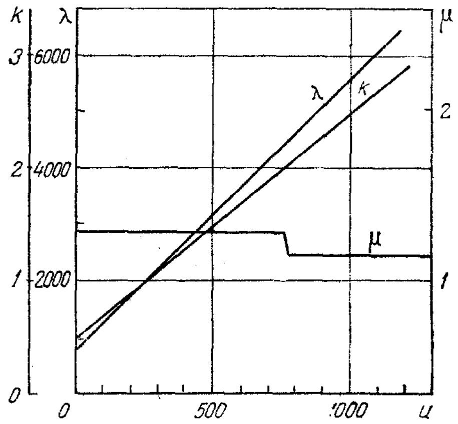
As control function we adopt $I = I(\omega, \alpha_V)$, where ω is the current frequency in the inductor; α_V is the internal heat-transfer coefficient between the phases which depends on the size of the pellets. The conditions of control with the specified speed of the gas and the geometric parameters of the installation are characterized by the value of $\varkappa$, and we will examine two cases: $\varkappa = 0$ (ideal heat insulation of the active region) and $\varkappa = 15$ W/m$^2\cdot$°C.

We will characterize the effect of control 1) by the function $\bar{w}(I)$ and also by the extremal values of the temperature $w|_{z=H}$ (w_{max} and w_{min}); 2) by the temperature and the position of the maximally heated point of the charge (u_{max}, r_m, z_m); 3) by the efficiency η of the installation.

In calculating the efficiency, we proceed from the diagram of the energy balance presented in Fig. 2, which is correct since the process is steady.

The energy accumulated by the gas is

$$\varepsilon_1 = 2\pi \int_0^R (c_g \rho_g v_g \varepsilon w)|_{z=H} r\,dr - 2\pi \int_0^R (c_g \rho_g v_g \varepsilon w)|_{z=0} r\,dr,$$



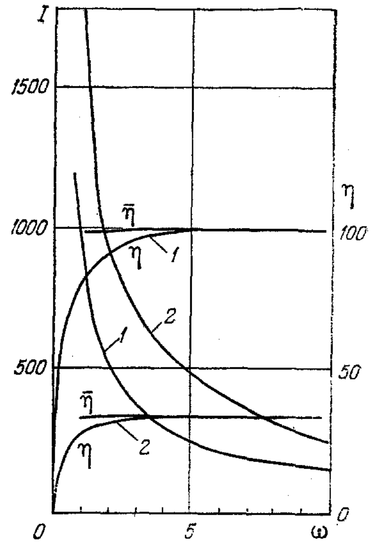


Fig. 3 Fig. 4

Fig. 3. Dependence of the effective characteristics of the medium on the temperature: k(u), W/m·°C; λ(u), $\Omega^{-1}\cdot m^{-1}$; u, °C.

Fig. 4. Dependences of the current intensity in the inductor and of the efficiency of the installation on the frequency of the current: 1) $\varkappa = 0$, $\alpha_V = 10^4$; 2) 15 and 10^4. I, A; η, %; ω, kHz.

the energy lost through heat transfer of the active medium with the environment is

$$\varepsilon_2 = 2\pi R \int_0^H k \frac{\partial u}{\partial r}\bigg|_{r=R} dz,$$

the losses in the inductor are $\varepsilon_3 = 0.5R_iI^2$, where the resistance of the inductor is $R_i = \rho L_i/S_i = 2.484\cdot10^{-3}$ Ω (ρ is the resistivity of copper; L_i is the total length of the inductor bus; S_i is its cross section).

Obviously, the energy release in the charge is

$$\varepsilon_4 \equiv 2\pi \int_0^H dz \int_0^R q(r, z)\, r dr = \varepsilon_1 + \varepsilon_2.$$

Thus,

$$\eta = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}\cdot 100\ \% = \frac{\varepsilon_1}{\varepsilon_3 + \varepsilon_4}\cdot 100\ \%.$$

Numerical experiments were carried out for an installation with the parameters R = 0.125 m, a = 0.45 m, H = 1.15 m, L = 0.7 m, h = 0.5 m, and D = 0.2 m (see Fig. 1). The dependences k(u), μ(u), and λ(u) obtained by graphic interpolation of the experimental data are shown in Fig. 3. In the system (1) ε = 0.4; $v_c = 0$; $c_c\rho_c = 3.588\cdot10^6$ W/m³·°C; $v_g = 1.019$ m/sec; $c_g\rho_g = 560$ W/m³·°C; $w_g^0 = 500$°C; δ = 0; $u_c^0 = 20$°C; $u_0 = 40$°C.

The values of the outlet characteristics of the heated gas in dependence on the variable parameters are presented in Table 1, from which it can be seen that in the mentioned range α_V has little influence on the effect of heating.

Figure 4 expresses the dependences I(ω) and η(ω) for the variants with $\varkappa = 0$ and $\varkappa \neq 0$. It may be noted that with increasing frequency of the current in the inductor $\eta \to \bar{\eta}$, i.e., the efficiency when there are no losses in the inductor. Already at a frequency of 2.4 kHz η and $\bar{\eta}$ are fairly close to each other. Also noticeable is the substantial influence of the level of heat insulation ϰ on the controlling and output characteristics of the installation.

Figure 5 shows the density distribution of the heat sources characteristic of all variants and the behavior of the temperature fields when ω = 2.4 kHz. It can be seen that in distinction to heating in a purely metallic medium, the skin effect is very slight. Correspond-

TABLE 1. Input and Output Parameters of the Operation of the Installation in Different Regimes

ω, kHz	α_V, W/m$^2\cdot$°C	$\varkappa$, W/m$^2\cdot$°C	I, A	$\bar{w}$, °C	Gas w_{max}, °C	Gas r, m	Gas w_{min}, °C	Gas r, m	Charge u_{max}, °C	Charge r, m	Charge z, m	$\bar{\eta}$, %	η, %
1	10^3	0	1035	985	1000	0,125	953	0	1078	0,125	0,767	100	78,2
1	10^4	0	1066	985	990	0,125	974	0	1001	0,125	0,894	100	77,2
1	10^3	15	1688	960	1023	0	877	0,125	1326	0,09	0,639	31,8	26,1
1	10^4	15	1721	947	1073	0	834	0,125	1188	0,09	0,767	33,1	26,6
2,4	10^3	0	440	975	990	0,125	942	0	1067	0,125	0,767	100	92,0
2,4	10^4	0	423	912	916	0,125	901	0	927	0,125	0,894	100	92,2
2,4	10^3	15	725	948	1011	0	865	0,125	1303	0,09	0,639	31,3	30,1
2,4	10^4	15	721	923	1048	0	810	0,125	1157	0,09	0,767	32,4	31,1
10	10^3	0	150	973	989	0,125	940	0	1060	0,125	0,767	100	100
10	10^4	0	143	952	957	0,125	940	0	966	0,125	0,894	100	100
10	10^3	15	264	954	1014	0	875	0,125	1282	0,09	0,639	30,4	30,4
10	10^4	15	252	930	1054	0	819	0,125	1178	0,09	0,767	31,1	31,1

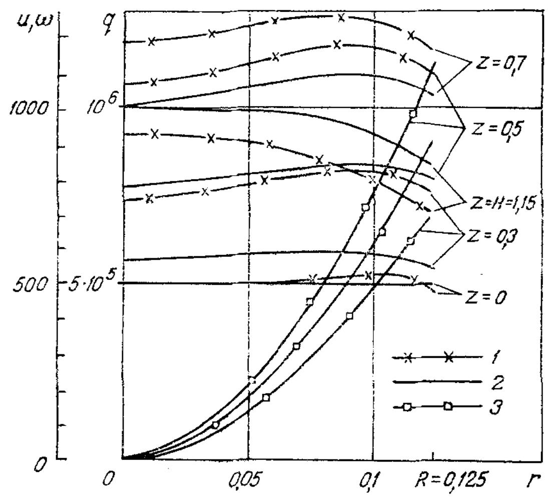


Fig. 5. Dependences of the temperature of the charge (1), of the gas temperature (2), of the density of the heat sources (W/m³) (3) on the radius for different z; u, w, °C; r, m.

ingly, with the current intensities chosen by us the medium is heated through to the required temperature (∼1000°C).

It may also be noted that in the specified range of values of the parameter of internal heat transfer α_V the heat transfer from the charge to the gas is very intense so that the temperature curves of the gas and of the charge do not differ very much, and they naturally approach each other with increasing α_V.

The results presented here and in Table 1 permit the assumption that the longitudinal dimensions of the heater may be reduced without loss of heating effect (with $\varkappa \neq 0$ there is a marked maximum of the temperature curves which is situated at some distance from the boundary of the working zone. (When $\varkappa = 0$, the behavior of the temperature curves is closer to monotonic.)

Thus the mathematical modeling of heating gas in a special heater, carried out on the basis of the solution of the problem of control, makes it possible to evaluate the effectiveness of the installation under examination, which may also be an element of a reduction set, and it is also possible to choose the optimum operating regime of the heater.

In conclusion the authors express their gratitude to Academician A. N. Tikhonov for his useful review.

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SOME GENERALIZATIONS OF THE THEORY OF OPTIMIZATION OF CASCADE THERMOELECTRIC COOLING UNITS

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UDC 537.322

The optimal conditions are generalized, taking account of the irreversible losses accompanying thermoelectric cooling.

Earlier [1], the problem of optimization of a thermoelectric cooling unit was solved for the case when there are no irreversible losses associated with nonideality of the electrical contacts and external heat inflows. The conditions for a minimum of the functional $\mu = Q_h/Q_c$ or the equivalent additive functional

$$J=\sum_{k=1}^{N}\ln q_{1m}^{k}-\sum_{k=1}^{N}\ln q_{0m}^{k} \tag{1}$$

were obtained, taking account of the temperature dependence of the parameters of the thermoelectric materials.

The mathematical model proposed in [1] is now corrected for the case when the thermoelement junctions are of finite electrical resistance. The expressions for the heat-flux densities at the boundaries of the k-th cascade take the form

$$q_{0m}^{k}=0.5\,[q_n(x_k^-)+q_p(x_k^-)+(i_n^k+i_p^k)\,R_c], \tag{2}$$

$$q_{1m}^{k}=0.5\,[q_n(x_{k-1}^+)+q_p(x_{k-1}^+)-(i_n^k+i_p^k)\,R_c]. \tag{3}$$

All the remaining relations in [1] are retained, but the derivatives $\partial J/\partial i_n^{k,p}$ in the equations for the optimal current densities are no longer equal to zero. Taking account of Eqs. (1)-(3), it is found that in the general case

$$\frac{\partial J}{\partial i_{n,p}^{k}}=-\beta R_c\left(\frac{1}{q_{0m}^{k}}+\frac{1}{q_{1m}^{k}}\right), \tag{4}$$

where $\beta = 1$ when $s_n^k = s_p^k(i_n^k = i_p^k)$ and $\beta = 1/2$ when the parameters s_n and s_p vary independently.

The appropriate analysis shows that when $R_c \neq 0$ and there are no constraints on the longitudinal dimension of the apparatus, the coordinates x_k, k = 1, ..., N, must be indeterminably large, i.e., there is no optimal sequence l_k. In practice, this means that the lengths of all the thermoelements must be equal to some limitingly large dimensions compatible with the specified size of the device, and hence the condition in Eq. (14) of [1] may be eliminated, as before.

Now consider the case when there are heat inflows from the surrounding medium. Heat transfer occurs on the free part of the heat-transfer surface and on the side surfaces of the cascades. Below, no account is taken of the lateral heat inflows; instead, it is assumed

Odessa Technological Institute of the Refrigeration Industry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 6, pp. 977-983, December, 1984. Original article submitted June 2, 1983.

 0022-0841/84/4706-1454$08.50